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Nonequilibrium Real Time Green's Functions and the Condition of Weakening of Initial Correlation

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A new method is given to calculate real-time Green's functions in nonequilibrium from the hierarchy of equations of motion in connection with the boundary condition of weakening of initial correlations. The way of deriving a generalized quantum Boltzmann equation is shown.

KEY WORDS: Nonequilibrium; Green's function; Boltzmann equation.

1. INTRODUCTION

The nonequilibrium properties of a quantum many-particle system can be described by the one-particle Green's function $g_1(1, 1')$ defined by[1]

$$g_1(1, 1') = \frac{1}{i} \operatorname{Tr}\{\rho T[\psi(1)\,\psi^+(1')]\} \tag{1}$$

where ρ is the unknown nonequilibrium density operator and T represents the Wick time ordering operation. The notation means $\mathbf{1} = \mathbf{r}_1, t_1$, etc. In addition to (1) the correlation functions are

$$g_1^>(1, 1') = \frac{1}{i} \operatorname{Tr}\{\rho\psi(1)\,\psi^+(1')\} \tag{2a}$$

$$g_1^<(1, 1') = \pm\frac{1}{i} \operatorname{Tr}\{\rho\psi^+(1')\,\psi(1)\} \tag{2b}$$

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The connection to the one-particle density matrix ρ_1 is given by

$$\rho_1(\mathbf{r}_1, \mathbf{r}_1'; t) = \pm i g_1^<(1, 1')_{|t_1 = t_1' = t} \tag{3}$$

In order to determine the Green's function we start from the well-known hierarchy of equations of motion for the real-time Green's functions, given by Martin and Schwinger.[1] The first equations are

$$\left[i \frac{\partial}{\partial t_1} + \frac{\nabla_1^2}{2m} - U(1) \right] g_1(1, 1') = \delta(1-1') \pm i \int d2\, V(1, 2)\, g_2(12, 1'2^+) \tag{4}$$

$$\left[i \frac{\partial}{\partial t_1} + \frac{\nabla_1^2}{2m} - U(1) \right] g_2(12, 1'2') = \delta(1-1')\, g_1(2, 2') \pm \delta(1-2')\, g_1(2, 1') \pm i \int d3\, V(1, 3)\, g_3(123, 1'2'3^+) \tag{5}$$

where $g_n(12 \cdots n,\ 1'2' \cdots n')$ is the n-particle Green's function, $V(1, 2) = V(\mathbf{r}_1 - \mathbf{r}_2)\, \delta(t_1 - t_2)$ is the two-body potential, and $U(1)$ is the external potential. These equations are very general, which means they do not depend on the kind of averaging. Therefore boundary conditions are necessary to obtain solutions for a special kind of averaging. For example, in the case of thermodynamic equilibrium, the system is described by the grand canonical density operator and the solutions are determined by the KMS condition for the imaginary-time domain.[1,2] For the one-particle Green's function one has

$$G(1, 1')_{|t_1 = 0} = \pm e^{\beta\mu} G(1, 1')_{|t_1 = -i\beta} \tag{6}$$

Here μ is the chemical potential, and $\beta = 1/k_B T$. The G denotes the Green's function defined for imaginary times.

In nonequilibrium situations, the KMS condition is not applicable. Up to now there exist two possibilities for the determination of nonequilibrium Green's functions:

(i) Kadanoff and Baym have performed an analytic continuation of the imaginary-time Green's functions to their real-time counterparts, assuming the system initially to be in thermodynamic equilibrium.[2]

(ii) In order to deal directly with real-time Green's functions in nonequilibrium situations, Keldysh introduced a diagrammatic perturbation scheme in order to evaluate the Green's functions in the interaction picture.[3] The technique is based on the use of a closed contour of chronological and antichronological time ordering.

The aim of this paper is to follow the consequent way of equations of motion for the determination of real-time Green's functions in nonequilibrium. A new method then is given to calculate real-time Green's functions directly from the hierarchy of equations of motion using the usual boundary condition of kinetic theory. This method is demonstrated for the simple case of binary collision approximation deriving the quantum Boltzmann equation. The generalization of the Boltzmann equation which takes into account medium-dependent three-particle scattering is discussed in the last section. A generalization of the Boltzmann equation taking into account phase space occupation was also given by Boercker and Dufty,[12] who started from the BBGKY hierarchy.

2. BOUNDARY CONDITION AND BINARY COLLISION APPROXIMATION

It is known that Green's functions are powerful quantities for the description of a many-particle system. We will determine real-time Green's functions in connection with the Bogoljubov condition of the total weakening of the initial correlation. That means we will solve the equations of motion with a boundary condition which reads for instance for the time-specialized Green's function describing the propagation of two particles in a many-particle system:

$$\lim_{t_1 \to -\infty} g_2(12, 1'2')\Big|_{\substack{t_1 = t_2 \\ t_1' = t_2' = t_1^{\pm}}} = g_1(1, 1')\, g_1(2, 2') \pm g_1(1, 2)\, g_1(2, 1') \tag{7}$$

Here the two possibilities for the case of equal times are taken into account by the limit $t_1' = t_1^{\pm} = t_1 \pm \varepsilon$. The boundary condition (7) is appropriate for situations in which long-time correlations and precollision spatial correlations are negligible. In this case the nonequilibrium properties of the system can be described by the one-particle Green's function. In systems with bound states and large-scale fluctuations the Bogoljubov condition of the form (7) is insufficient and must be generalized.[5 7] In the following we will consider such systems for which the condition (7) is possible.

In order to derive a kinetic equation for $g_1(1, 1')$ it is necessary to get solutions for $g_2(12, 1'2')$ which satisfy the Bogoljubov condition (7). To demonstrate this concept we consider first the simple case of the binary collision approximation. In this approximation g_2 can be determined by the following equation, which arises from the Martin–Schwinger hierarchy by neglecting higher than two-particle collisions and self-energy corrections to the one-particle Green's function:

$$\left[i\frac{\partial}{\partial t_1} + \frac{\nabla_1^2}{2m} - U(1)\right]\left[i\frac{\partial}{\partial t_2} + \frac{\nabla_2^2}{2m} - U(2)\right]\{g_2(12, 1'2') - g_1^0(1, 1')\, g_1^0(2, 2')$$
$$\mp g_1^0(1, 2')\, g_1^0(2, 1')\} = iV(1, 2)\, g_2(12, 1'2') \tag{8}$$

where g_1^0 is the free one-particle Green's function satisfying

$$\left[i\frac{\partial}{\partial t_1}+\frac{\nabla_1^2}{2m}-U(1)\right]g_1^0(1,1')=\delta(1-1')$$

A solution, satisfying the Bogoljubov condition (7), follows directly from (8) and can be represented in the form $(t_1=t_2,\ t_1'=t_2')$

$$\begin{aligned} g_2^L(12,1'2') &= \mathscr{G}_2^{HF}(12,1'2')+i\int_{-\infty}^{\infty} d\bar{1}\,d\bar{2}\,V(\bar{1},\bar{2}) \\ &\times \{\mathscr{G}_2(12,\bar{1}\bar{2})\,g_2^L(\bar{1}\bar{2},1'2')-\mathscr{G}_2^<(12,\bar{1}\bar{2})\,g_2^{L>}(\bar{1}\bar{2},1'2')\} \end{aligned} \tag{9}$$

(here we have denoted the binary collision approximation by the superscript L), where

$$\mathscr{G}_2^{HF}(12,1'2')=g_1^0(1,1')\,g_1^0(22')\pm g_1^0(1,2')\,g_1^0(2,1') \tag{10}$$

The function $\mathscr{G}_2(12,1'2')$ is given by (10) without the exchange term. In (8) and (9) the approximation of free particles is used for the one-particle Green's function. It is easily proved that (9) is a solution of Eq. (8). The fact that the boundary condition (7) is fulfilled can be seen from (9) if one splits up the region of time integration for a special time order of t_1 and t_1':

$$\int_{-\infty}^{\infty} d\bar{t}=\int_{-\infty}^{t_1} d\bar{t}_1+\int_{t_1}^{t_1'} d\bar{t}_1+\int_{t_1'}^{\infty} d\bar{t}_1 \qquad (\text{for } t_1<t_1')$$

For the further considerations it is useful to introduce a T matrix by the definition

$$\begin{aligned} \langle \mathbf{r}_1\mathbf{r}_2|\,T_{12}(t,t')\,|\mathbf{r}_1'\mathbf{r}_2'\rangle &= V(\mathbf{r}_1-\mathbf{r}_2)\,\delta(\mathbf{r}_1-\mathbf{r}_1')\,\delta(\mathbf{r}_2-\mathbf{r}_2')\,\delta(t-t') \\ &+ iV(\mathbf{r}_1-\mathbf{r}_2)\,\tilde{g}_2^L(\mathbf{r}_1\mathbf{r}_2t,\mathbf{r}_1'\mathbf{r}_2't')\,V(\mathbf{r}_1'-\mathbf{r}_2') \end{aligned} \tag{11}$$

where $\tilde{g}_2^L$ is the two-particle Green's function without the exchange contribution. Using (9) we get the following equation for T_{12}

$$\begin{aligned} \langle \mathbf{r}_1\mathbf{r}_2|\,T_{12}(t,t')\,|\mathbf{r}_1'\mathbf{r}_2'\rangle &= V(\mathbf{r}_1-\mathbf{r}_2)\,\delta(\mathbf{r}_1-\mathbf{r}_1')\,\delta(\mathbf{r}_2-\mathbf{r}_2')\,\delta(t-t') \\ &+ i\int_{-\infty}^{\infty} d\bar{t}\,d\bar{\mathbf{r}}_1\,d\bar{\mathbf{r}}_2\,V(\mathbf{r}_1-\mathbf{r}_2)\,\mathscr{G}_2(\mathbf{r}_1\mathbf{r}_2t,\bar{\mathbf{r}}_1\bar{\mathbf{r}}_2\bar{t}) \\ &\times \langle \bar{\mathbf{r}}_1\bar{\mathbf{r}}_2|\,T_{12}(\bar{t},t')\,|\mathbf{r}_1'\mathbf{r}_2'\rangle \\ &- i\int_{-\infty}^{\infty} d\bar{t}\,d\bar{\mathbf{r}}_1\,d\bar{\mathbf{r}}_2\,V(\mathbf{r}_1-\mathbf{r}_2)\,\mathscr{G}_2^<(\mathbf{r}_1\mathbf{r}_2t,\bar{\mathbf{r}}_1\bar{\mathbf{r}}_2\bar{t}) \\ &\times \langle \bar{\mathbf{r}}_1\bar{\mathbf{r}}_2|\,T_{12}^>(\bar{t},t')\,|\mathbf{r}_1'\mathbf{r}_2'\rangle \end{aligned} \tag{12}$$

It can be shown that Eq. (12) is the many-particle version of the T matrix of ordinary scattering theory, which takes into account the Pauli blocking in the two-particle scattering in nonequilibrium situations. Especially the optical theorem follows from (12) which already was derived by Baerwinkel[8] on the basis of the Kadanoff–Baym technique.

Let us now consider the right-hand side of Eq. (4) for $g_1(1, 1')$ in the case $t_1 < t_1'$. Using the symmetrized (antisymmetrized) T matrix we find

$$\int d2\, V(1,2)\, g_2^L(12, 1'2^+) = \int_{-\infty}^{\infty} d2 d\bar{1} d\bar{2} \{\langle 12|\, T_{12}\, |\bar{1}\bar{2}\rangle^{\pm}\, \mathscr{G}_2(\bar{1}\bar{2}, 1'2^+) - \langle 12|\, T_{12}^{\lessgtr}\, |\bar{1}\bar{2}\rangle^{\pm}\, \mathscr{G}_2^{>}(\bar{1}\bar{2}, 1'2^+)\} \tag{13}$$

with

$$\langle 12|\, T_{12}\, |1'2'\rangle^{\pm} = \langle \mathbf{r}_1\mathbf{r}_2|\, T_{12}(t_1, t_1')\, |\mathbf{r}_1'\mathbf{r}_2'\rangle^{\pm}\, \delta(t_1 - t_2)\, \delta(t_1' - t_2')$$

If we replace the free by the full one-particle Green's function up to higher orders and if we introduce the self-energy in binary collision approximation,

$$\Sigma_{(2)}(1, 1') = \pm i \int d2 d\bar{2}\, \langle 12|\, T_{12}\, |1'\bar{2}\rangle^{\pm}\, g_1(\bar{2}, 2^+) \tag{14}$$

we get the structure of the well-known equation of motion for $g_1^<(1, 1')$ given by Kadanoff and Baym,[2]

$$\left[i\frac{\partial}{\partial t_1} + \frac{\nabla_1^2}{2m} - U(1)\right] g_1^<(1, 1') = \int d\bar{\mathbf{r}}_1\, \Sigma^{\mathrm{HF}}(\mathbf{r}_1\bar{\mathbf{r}}_1, t_1)\, g_1^<(\bar{\mathbf{r}}_1 t_1, \mathbf{r}_1' t_1') + \int_{-\infty}^{t_1} d\bar{1}\, \Sigma_{(2)}^{>}(1\bar{1})\, g_1^<(\bar{1}1') + \int_{t_1}^{t_1'} d\bar{1}\, \Sigma_{(2)}^{<}(1\bar{1})\, g_1^<(\bar{1}1') + \int_{t_1'}^{-\infty} d\bar{1}\, \Sigma_{(2)}^{<}(1\bar{1})\, g_1^>(\bar{1}1') \tag{15}$$

which in our case is a direct consequence of the boundary condition (7).

We want to underline that the boundary condition for g_2 leads to a corresponding condition for the total self-energy,

$$\lim_{t_1 \to -\infty} \int d2\, V(1,2)\, g_2(12, 1'2^+)_{|t_1' = t_1^+} = \int d2\, V(1,2)\{g_1(1,1')\, g_1(2, 2^+) \pm g_1(1, 2^+)\, g_1(2, 1')\} = \mp i \int d\bar{\mathbf{r}}_1\, \Sigma^{\mathrm{HF}}(\mathbf{r}_1\bar{\mathbf{r}}_1, t_1)\, g_1^<(\bar{\mathbf{r}}_1 t_1, \mathbf{r}_1' t_1') \tag{16}$$

Using (16) in the equation of motion (4) for g_1 one can derive the general form of the Kadanoff–Baym kinetic equation which is given by (15) replacing $\Sigma_{(2)}$ by the total self-energy. That means the Eq. (15) is valid for any approximation of the self-energy.

In the further approach one can follow the techniques used in Ref. (2). Taking into account the optical theorem for the real-time T matrix we finally get the quantum Boltzmann equation in binary collision approximation.

3. GENERALIZATION OF THE BOLTZMANN EQUATION

In order to take into account higher approximations with respect to the particle correlation, a cluster expansion for g_2 must be used at the right-hand side of Eq. (4). The main line for doing this in the presented theory should be briefly explained.

From the hierarchy of equations of motion, the following cluster expansion was derived[9]:

$$g_2(12, 1'2') = g_2^L(12, 1'2') \pm \int d3d\bar{3}\; g_1^{0-1}(3, \bar{3})$$

$$\times\; [g_3^L(12\bar{3}, 1'2'3^+) - g_2^L(12, 1'2')\, g_1^0(\bar{3}, 3^+)] \qquad (17)$$

The first term, g_2^L, is the contribution of the binary collision approximation, determined by Eq. (9). The second term describes the influence of the three-particle correlations.

We want to underline that all times are different in (17). In Eq. (4) a special case of the two-particle Green's function must be used depending only on two time arguments. To get the similar dependence on two time arguments in the contribution of three-particle correlations, we consider the three-particle Green's function g_3^L for the special case $t_1 = t_2 = t_3$ and (for simplicity) $t'_1 = t'_2 = t'_3$. The Bogoljubov condition takes the form

$$\lim_{t_1 \to -\infty} g_{123}(t_1, t'_1)_{|t'_1 = t_1^{\pm}} = g_1(t_1, t'_1)\, g_2(t_1, t'_1)\, g_3(t_1, t'_1)$$

$$\pm \text{ exchange terms} \qquad (18)$$

Then the following solution can be derived approximately for the considered time specialization in the case of three-particle Ladder approximation[9]:

$$g^L_{123}(t_1, t'_1) = \mathscr{G}_{123}(t_1, t'_1) \pm \text{exchange terms}$$
$$+ i^2 \int_{-\infty}^{+\infty} d\bar{t}_1\, \mathscr{G}_{123}(t_1, \bar{t}_1)\, v^{\text{eff}}_{123}(\bar{t}_1)\, g^L_{123}(\bar{t}_1, t'_1)$$
$$- i^2 \int_{-\infty}^{+\infty} d\bar{t}_1\, \mathscr{G}^{<}_{123}(t_1, \bar{t}_1)\, v^{\text{eff}}_{123}(\bar{t}_1)\, g^{L>}_{123}(\bar{t}_1, t'_1) \qquad (19)$$

where we have for free particles

$$\mathscr{G}_{123}(t, t') = g^0_1(t, t')\, g^0_2(t, t')\, g^0_3(t, t')$$

In (19) an effective three-particle potential was introduced which in lowest order of the one-particle correlation functions $g^{0<}_i(t)$ $(i = 1, 2, 3)$ is given by

$$\begin{aligned} v^{\text{eff}}_{123}(t) &= v^{\text{eff}}_{12}(t) + v^{\text{eff}}_{13}(t) + v^{\text{eff}}_{23}(t) \\ &= [1 - ig^{0<}_3(t, t)]\, V_{12} + [1 - ig^{0<}_2(t, t)]\, V_{13} \\ &\quad + [1 - ig^{0<}_1(t, t)]\, V_{23} \end{aligned} \qquad (20)$$

In (18), (19), and (20) a matrix notation for the space variables was used in which multiplication involves integrating the coordinate matrix indices over all space.

It can be seen that the pure three-particle potential $V_{123} = V_{12} + V_{13} + V_{23}$ follows from (20) in the low-density limit if the correlation functions are negligible.

It is useful to define a T matrix by

$$T_{123}(t, t') = v^{\text{eff}}_{123}(t)\, \delta(t - t')$$
$$+ i^2 v^{\text{eff}}_{123}(t)\, \tilde{g}^L_{123}(t, t')\, v^{\text{eff}}_{123}(t') \qquad (21)$$

(sometimes we will use a modified T matrix given by $T^{10}_{123}(t, t') = (v^{\text{eff}}_{12} + v^{\text{eff}}_{13}) + i^2(v^{\text{eff}}_{12} + v^{\text{eff}}_{13})\, \tilde{g}^L_{123} v^{\text{eff}}_{123}$) Here $\tilde{g}^L_{123}$ is the three-particle Green's function without the exchange contribution. Using (19) the following equation for T_{123} can be derived from (21):

$$T_{123}(t_1, t'_1) = v^{\text{eff}}_{123}(t_1)\, \delta(t_1 - t'_1)$$
$$+ i^2 \int_{-\infty}^{+\infty} d\bar{t}_1\, v^{\text{eff}}_{123}(t_1)\, \mathscr{G}_{123}(t_1, \bar{t}_1)\, T_{123}(\bar{t}_1, t'_1)$$
$$- i^2 \int_{-\infty}^{+\infty} d\bar{t}_1\, v^{\text{eff}}_{123}(t_1)\, \mathscr{G}^{<}_{123}(t_1, \bar{t}_1)\, T^{>}_{123}(\bar{t}_1, t'_1) \qquad (22)$$

A T matrix formulation is presented by Eq. (22) which describes three-particle scattering in a nonequilibrium many-body medium. Similar equations are given in Ref. 10 for the case of thermodynamic equilibrium using (imaginary-time) diagram techniques. Now we insert the cluster expansion (17) in Eq. (4) for $g_1^<$ specializing the time arguments in the right manner. Introducing the self-energy we get the known structure of Eq. (15) replacing $\Sigma_{(2)}$ by $\Sigma_{(3)}$, which is given by the expansion

$$\begin{aligned}\Sigma_{(3)}(1,1') = &\pm i \int d2 d\bar{2}\, \langle 12|\, T_{12}\, |1'\bar{2}\rangle^{\pm}\, g_1(\bar{2}2^+)\\ &+ \frac{1}{2}(\pm i)^2 \int d2\, d3\, d\bar{2}\, d\bar{3} \{\langle 123|\, T^{10}_{123}\, |1'\bar{2}\bar{3}\rangle^{\pm}\, g_1(\bar{2}2^+)\, g_1(\bar{3}3^+)\\ &- R(1,2,3;1'2^+3^+)\}\end{aligned} \tag{23}$$

In the derivation of (23) the functions g^0 are replaced by g. With R we have taken into account the corresponding self-energy contributions and the disconnected three-particle terms.[9]

Finally we will give the generalization of the Boltzmann equation for the one-particle distribution function which can be derived from Eq. (15) with the self-energy (23) in three-particle collision approximation. Following the techniques used in Ref. 2 we get for the spatially homogeneous case

$$\left\{\frac{\partial}{\partial T} - [\nabla_{\mathbf{R}} U(\mathbf{R}, T) \cdot \nabla_{\mathbf{p}}]\right\} f(\mathbf{p}, T) = I_2(\mathbf{p}) + I_3(\mathbf{p}) + I_R(\mathbf{p}) \tag{24}$$

where $-\nabla_{\mathbf{R}} U(\mathbf{R}, T) = F(T)$ is the external force field. The two-particle collision integral is given by

$$\begin{aligned}I_2(\mathbf{p}) = &\frac{1}{V}\int \frac{d\bar{\mathbf{p}}_1}{(2\pi)^3}\frac{d\mathbf{p}_2}{(2\pi)^3}\frac{d\bar{\mathbf{p}}_2}{(2\pi)^3}\frac{1}{2!}\,|\langle \mathbf{p}_1\mathbf{p}_2|\, \mathscr{T}_{12}(E_{12}+i0, T)\, |\bar{\mathbf{p}}_1\bar{\mathbf{p}}_2\rangle^{\pm}|^2\\ &\times 2\pi\delta(E_{12}-\bar{E}_{12})\{\bar{f}_1\bar{f}_2(1\pm f_1)(1\pm f_2) - (1\pm\bar{f}_1)(1\pm\bar{f}_2) f_1 f_2\}\end{aligned} \tag{25}$$

For the three-particle collision integral we get the expression

$$\begin{aligned}I_3(\mathbf{p}) = &\frac{1}{2V}\int \frac{d\mathbf{p}_2}{(2\pi)^3}\frac{d\mathbf{p}_3}{(2\pi)^3}\frac{d\bar{\mathbf{p}}_1}{(2\pi)^3}\frac{d\bar{\mathbf{p}}_2}{(2\pi)^3}\frac{d\bar{\mathbf{p}}_3}{(2\pi)^3}\, 2\pi\delta(E_{123}-\bar{E}_{123})\\ &\times \frac{1}{3!}\,|\langle \mathbf{p}_1\mathbf{p}_2\mathbf{p}_3|\, \mathscr{T}_{123}(E_{123}+i0, T)\, |\bar{\mathbf{p}}_1\bar{\mathbf{p}}_2\bar{\mathbf{p}}_3\rangle^{\pm}|^2{}_{|\text{connected}}\\ &\times \{\bar{f}_1\bar{f}_2\bar{f}_3(1\pm f_1)(1\pm f_2)(1\pm f_3) - (1\pm\bar{f}_1)(1\pm\bar{f}_2)(1\pm\bar{f}_3) f_1 f_2 f_3\}\end{aligned} \tag{26}$$

where $f_1 = f(\mathbf{p}_1, T)$; $\bar{f}_1 = f(\bar{\mathbf{p}}_1, T)$, etc. are the momentum distribution functions and E_{12}, E_{123} are the scattering energies of free particles. It can be seen that the collision integrals are expressed by retarded T matrices which describe the scattering in the nonequilibrium many-particle system.

Because of the integration over p_2 and p_3 we can use in I_3 the T matrix $\mathscr{T}_{123}$ instead of $\mathscr{T}^{10}_{123}$ [cf. Eq. (23)].

$\mathscr{T}_{123}(z, T)$ is given by [9] (operator notation)

$$\mathscr{T}_{123}(z, T) = v^{\text{eff}}_{123}(T) + v^{\text{eff}}_{123}(T)\, \mathscr{G}_{123}(z, T)\, \mathscr{T}_{123}(z, T) \tag{27}$$

where

$$\mathscr{G}_{123}(z, T) = \int \frac{d\omega}{2\pi} \frac{\mathscr{G}^{>}_{123}(\omega, T) - \mathscr{G}^{<}_{123}(\omega, T)}{z - \omega} \tag{28}$$

With $\mathscr{T}_{123}$ the contribution of the medium dependent three-particle scattering to the corresponding collision integral (26) is given. Especially a time-dependent effective three-particle potential appears in the T matrix equation (27).

The term I_R arises from self-energy corrections R. To lowest approximation this term is found to be

$$\begin{aligned} I_R = -\frac{1}{V}\int \frac{d\mathbf{p}_2}{(2\pi)^3}\frac{d\mathbf{p}_3}{(2\pi)^3} \langle \mathbf{p}_1\mathbf{p}_2\mathbf{p}_3 | \,[V_{12}, \Omega_{12}(\Omega_{13} f_1 f_2 f_3 \Omega^{+}_{13} \\ - f_1 f_2 f_3)\,\Omega^{+}_{12} + \Omega_{12}(\Omega_{23} f_1 f_2 f_3 \Omega^{+}_{23} - f_1 f_2 f_3)\,\Omega^{+}_{12}]\, | p_1 p_2 p_3 \rangle \end{aligned} \tag{29}$$

Here we have used an operator notation. The functions Ω correspond to the Moeller operator of scattering theory. They are connected with the two-particle T matrix by

$$\mathscr{T}_{12}(E_{12} + i0) = V_{12}\Omega_{12}(E_{12} + i0) \tag{30}$$

A similar expression for I^R was also found by other authors.[5,7] From a principle point of view I_R is very important as it compensates for the successive binary collisions in the three-particle integral. Further it ensures the conservation of the energy in binary collision approximation. Therefore the kinetic equation (24) is suited to describe nonideal gases.[6]

It is the quantum mechanical generalization of the well-known kinetic equation for classical dense gases.[13,14]

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